

# Higher Mathematics

## Key Facts and Methods



Use this as part of your on-going revision by:-

1. Testing yourself
2. Asking someone to test you by reading the questions to you
3. Use the questions to jog your memory when you are stuck on a question from the revision questions booklet.

The questions are on the left-hand side of each page and the answers are on the right.

The person testing you does not need to have any mathematical knowledge. Some mathematical symbols have been written out in full to help them read them out loud.

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## National 5 Revision

What is needed to add or subtract fractions?		A common denominator
What order should you try to factorise?		1. Common factor 2. Difference of two squares 3. Trinomial (into two brackets)
How do you solve a quadratic equation?		Make it equal to zero and factorise OR Use the quadratic formula
What is the quadratic formula?		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (x equals minus b plus or minus the square root of b squared minus 4 a c all over 2 a)
What is $a^m \times a^n$ ? (a to the power of m times a to the power of n)		$a^{m+n}$ (a to the power of m plus n)
What is $a^m \div a^n$ ? (a to the power of m times a to the power of n?)		$a^{m-n}$ (a to the power of m minus n)
What is $(a^m)^n$ ? (a to the power of m all to the power of n?)		$a^{mn}$ (a to the power of mn (m times n))
Write $a^{-n}$ with a positive power. (a to the power of negative n)		$\frac{1}{a^n}$ (one over a to the power of n)
Write $a^{\frac{m}{n}}$ as a surd. (a to the power of m divided by n)		$\sqrt[n]{a^m}$ (The n <sup>th</sup> root of a to the power of m)
What is $a^0$ ? (a to the power of zero)		1
What is $a^1$ ? (a to the power of one)		a
What is $\sqrt{a} \times \sqrt{b}$ ? (root a times root b)		$\sqrt{ab}$ (root ab)
How do you rationalise the denominator of a fraction (e.g. $\frac{1}{\sqrt{a}}$ )? (one over root a)		Multiply by $\frac{\sqrt{a}}{\sqrt{a}}$ (root a over root a)
In what quadrants are the trig ratios positive?		1. All 2. Sin 3. Tan 4. Cos <div><div>S</div><div>A</div><div>T</div><div>C</div></div>
If $x$ is an acute angle, how do you find the equivalent angle in	Quadrant 2 (S)?	$180 - x$
	Quadrant 3 (T)?	$180 + x$
	Quadrant 4 (C)?	$360 - x$
How do you find the y-intercept of a graph?		Substitute $x = 0$ into the equation

How do you find the roots of a graph?	Substitute $y = 0$ into the equation and solve
What points need labelled on the graph (if they exist)?	1. Stationary points 2. Roots 3. y-intercept

Trigonometric Graphs and Exact Values		
How do you find the solution to $\sin x = 0$ , $\sin x = 1$ , $\sin x = -1$ , $\cos x = 0$ , $\cos x = 1$ and $\cos x = -1$ ?	Use the graph	
What are the solutions of $\sin x = 0$ ?	$0^\circ$ , $180^\circ$ and $360^\circ$ OR $0$ , $\pi$ and $2\pi$ radians ( $0$ , $\pi$ or $2\pi$ )	
What is the solution of $\sin x = 1$ ?	$90^\circ$ OR $\frac{\pi}{2}$ radians ( $\pi$ over $2$ )	
What is the solution of $\sin x = -1$ ?	$270^\circ$ OR $\frac{3\pi}{2}$ radians ( $3\pi$ over $2$ )	
What are the solutions of $\cos x = 0$ ?	$90^\circ$ , $270^\circ$ OR $\frac{\pi}{2}$ , $\frac{3\pi}{2}$	
What are the solutions of $\cos x = 1$ ?	$0^\circ$ and $360^\circ$ OR $0$ and $2\pi$ radians	
What is the solution of $\cos x = -1$ ?	$180^\circ$ OR $\pi$ radians	
How do you evaluate $\sin$ , $\cos$ or $\tan$ of $30^\circ$ ( $\frac{\pi}{6}$ radians), $45^\circ$ ( $\frac{\pi}{4}$ radians) or $60^\circ$ ( $\frac{\pi}{3}$ radians)?	Exact value triangles	
What is the value of	$\sin 30^\circ$ or $\sin\left(\frac{\pi}{6}\right)$ ?	$\frac{1}{2}$ (a half)
	$\cos 30^\circ$ or $\cos\left(\frac{\pi}{6}\right)$ ?	$\frac{\sqrt{3}}{2}$ (root 3 over 2)
	$\tan 30^\circ$ or $\tan\left(\frac{\pi}{6}\right)$ ?	$\frac{1}{\sqrt{3}}$ (1 over root 3)
	$\sin 45^\circ$ or $\sin\left(\frac{\pi}{4}\right)$ ?	$\frac{1}{\sqrt{2}}$ (1 over root 2)

	$\cos 45^\circ$ or $\cos\left(\frac{\pi}{4}\right)?$	$\frac{1}{\sqrt{2}}$
	$\tan 45^\circ$ or $\tan\left(\frac{\pi}{4}\right)?$	1
	$\sin 60^\circ$ or $\sin\left(\frac{\pi}{3}\right)?$	$\frac{\sqrt{3}}{2}$
	$\cos 60^\circ$ or $\cos\left(\frac{\pi}{3}\right)?$	$\frac{1}{2}$
	$\tan 60^\circ$ or $\tan\left(\frac{\pi}{3}\right)?$	$\sqrt{3}$ (root 3)
What is $180^\circ$ in radians?		$\pi$ (pi)
What is $360^\circ$ in radians?		$2\pi$ (2pi)
What is $90^\circ$ in radians?		$\frac{\pi}{2}$ (pi over 2)
What is $\pi$ radians in degrees?		$180^\circ$
What is $2\pi$ radians in degrees?		$360^\circ$
What is $\frac{\pi}{2}$ radians in degrees?		$90^\circ$
What are the maximum and minimum values for $\sin x$ and $\cos x$ ?		maximum = 1 minimum = -1

### Expressions and Formula 1.1 – Exponential and Logarithmic functions

Which two points does the graph of $y = a^x$ pass through? (a to the power of x)		(0, 1) and (1, a)
Which points does the graph of $y = \log_a x$ pass through? (log to the base a of x)		(1, 0) and (a, 1)
What is $\log_e x$ ? (log base e of x)		Natural log or $\ln x$
How do you write $y = \log_a x$ in exponential form? (log to the base a of x)		$x = a^y$
Laws of logs	$\log_a x + \log_a y = ?$ (log base a of x plus log base a of y)	$\log_a xy$ (log base a of x y)
	$\log_a x - \log_a y = ?$ (log base a of x plus log base a of y)	$\log_a \frac{x}{y}$ (log base a of x over y)
	$\log_a(x^n) = ?$ (log base a of x to the power of n)	$n \log_a x$ (n log base a of x)
	$\log_a 1 = ?$ (log to the base a of 1)	0
	$\log_a 0 = ?$ (log to the base a of 0)	1

How do you solve an equation where $x$ is a power? (e.g. $4^x = 10$ )	<ol style="list-style-type: none"> <li>1. Take logs of both sides</li> <li>2. Use laws of logs to find <math>x</math></li> </ol>
How do you solve log equations?	Rearrange to get single log expressions on both sides, then “cancel” to solve OR Rearrange to get logs expression on one side with a number on the other, then rewrite in exponential form
A straight line graph has $\log x$ or $\log y$ on the axes. How do you find the equation?	<ol style="list-style-type: none"> <li>1. Find the equation of the line using <math>y = mx + c</math></li> <li>2. Replace <math>x</math> and <math>y</math> with whatever is on the axes</li> <li>3. Use the logs of logs to rearrange</li> </ol>

<b>Expressions and Formula 1.2 – Compound Angle Formula and Wave Function</b>		
What is the formula for: (formula are on the formula list)	$\sin(A + B)?$	$\sin A \cos B + \cos A \sin B$
	$\sin(A - B)?$	$\sin A \cos B - \cos A \sin B$
	$\cos(A + B)?$	$\cos A \cos B - \sin A \sin B$
	$\cos(A - B)?$	$\cos A \cos B + \sin A \sin B$
What is the formula for: (formula are on the formula list)	$\sin 2A?$	$2 \sin A \cos A$
	$\cos 2A?$	$\cos^2 A - \sin^2 A$
		$2 \cos^2 A - 1$
		$1 - 2 \sin^2 A$
If you are given the exact value of the sine of an acute angle, how can you find cos or tan?	<ol style="list-style-type: none"> <li>1. Draw a right angled triangle</li> <li>2. Use Pythagoras to find the adjacent side</li> <li>3. Used SOH CAH TOA to find the other solutions</li> </ol>	
If you are given the exact value of the cosine of an acute angle, how can you find sin or tan?	<ol style="list-style-type: none"> <li>1. Draw a right angled triangle</li> <li>2. Use Pythagoras to find the opposite side</li> <li>3. Used SOH CAH TOA to find the other solutions</li> </ol>	
If you are given the exact value of the tangent of an acute angle, how can you find cos or sin?	<ol style="list-style-type: none"> <li>1. Draw a right angled triangle</li> <li>2. Use Pythagoras to find the hypotenuse</li> <li>3. Used SOH CAH TOA to find the other solutions</li> </ol>	
How do you solve “simple” trig equations (sin, cos or tan only)?	<ol style="list-style-type: none"> <li>1. Rearrange to get <math>\sin x = \dots</math></li> <li>2. Ignore negatives and use inverse to find the related acute angle</li> <li>3. Use <math>\frac{S}{T} \mid \frac{A}{C}</math> to find the other solution(s)</li> </ol>	

If you know the solutions to trig equation between $0^\circ$ and $360^\circ$ , how do you find other solutions?	Add or take away multiples of $360^\circ$
How do you solve double angle trig equations?	1. Replace the double angle with the appropriate formula 2. Factorise and solve for $x$
What should you check when answering trig equation questions?	Whether your answer is to be in degrees or radians
How do you rewrite $y = a\sin x + b\cos x$ in the form $y = k(\sin x \pm \alpha)$ or $y = k(\cos x \pm \alpha)$ ?	1. Expand the compound angle formula 2. Equate coefficients to find $k\sin\alpha$ and $k\cos\alpha$ 3. Solve $k^2 + a^2 + b^2$ 4. Solve $\tan\alpha = \frac{k\sin\alpha}{k\cos\alpha}$ for $\alpha$
How do you find the correct quadrant for $\alpha$ ?	1. See if $\sin$ , $\cos$ and $\tan$ are positive or negative 2. Use $\begin{array}{c c} \text{S} & \text{A} \\ \hline \text{T} & \text{C} \end{array}$
How do you find the maximum and / or minimum values of a wave function?	Maximum when $\sin x \pm \alpha$ or $\cos x \pm \alpha = 1$ Minimum when $\sin x \pm \alpha$ or $\cos x \pm \alpha = -1$
How do you solve $a\sin x + b\cos x = c$ ?	1. Rewrite LHS as a wave function 2. Solve as a trig equation

### Expressions and Formulae 1.3 – Functions, inverses and graphs of functions

What is the domain of a function?	The set of numbers which go INTO the function
What is the range of a function	The set of functions which come OUT of the function
If a function contains a square root, its domain is restricted. The expression under the square root is...	$\geq 0$ (greater than or equal to zero)
If a function includes dividing, its domain is restricted. The expressions in the denominator are...	$\neq 0$ (not equal to zero)

If $f(g(x)) = g(f(x)) = x$ , what is the connection between the functions $f$ and $g$ ? ( $f$ of $g$ of $x$ equals $g$ of $f$ of $x$ )	They are inverse functions
How do you find the inverse of $f(x)$ ? ( $f$ of $x$ )	1. Swap $f(x)$ for $x$ 2. Make $x$ the subject of the equation 3. Swap $x$ for $f^{-1}(x)$
When given the graph $y = f(x)$ , how do you find:	( $y$ equals $f$ of $x$ )
$y = f(x) + k$	Move the graph <b>up</b> by $k$ units
$y = f(x) - k$	Move the graph <b>down</b> by $k$ units
$y = kf(x)$	Stretch the graph vertically (up/ down) by a factor of $k$
$y = f(kx)$	Squash the graph horizontally (left/ right) by a factor of $k$
$y = f(x + k)$	Move the graph <b>left</b> by $k$ units
$y = f(x - k)$	Move the graph <b>right</b> by $k$ units
$y = -f(x)$	Reflect in the $x$ -axis (flipped vertically)
$y = f(-x)$	Reflect in the $y$ -axis (flipped horizontally)

### Expressions and Formulae 1.4 – Vectors

What is a vector?	An object with magnitude(size) and direction
What is a scalar?	An object with only magnitude
How do you find the magnitude of $\mathbf{u} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ ?	$ \mathbf{u}  = \sqrt{a^2 + b^2 + c^2}$ (magnitude of $\mathbf{u}$ is the square root of a squared plus $b$ squared plus $c$ squared)
What is the magnitude of a unit vector?	One
How do you find a unit vector parallel to a given vector?	1. Find the magnitude 2. Multiply the original vector by $\frac{1}{\text{magnitude}}$
How do you find the component form of the vector $\overrightarrow{AB}$ ?	$\mathbf{b} - \mathbf{a}$
How can $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ be written in terms of $\mathbf{i}$ , $\mathbf{j}$ , $\mathbf{k}$ ?	$a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$
If $\mathbf{u}$ is a vector, what route is described by the negative vector $-\mathbf{u}$ ?	It travels backwards along $\mathbf{u}$



If <b>u</b> and <b>v</b> are vectors, what route is described by <b>u + v</b> ?		Travel along <b>u</b> and then <b>v</b> when they lie nose to tail.
How do you show that vectors <b>u</b> and <b>v</b> are parallel?		Show that <b>u = kv</b> (that they are multiples of each other)
How do you show that vectors are collinear?		1. Show and <b>state</b> that they are parallel 2. Show and <b>state</b> they share a common point
How do you find point P that divides a vector in the ratio m:n?		1. $\mathbf{p} = \frac{1}{m+n} (n\mathbf{a} + m\mathbf{b})$ 2. Write <b>p</b> in coordinate form
What is the scalar product in: (Formula is on the formula list)	angle form?	$\mathbf{a} \cdot \mathbf{b} =  \mathbf{a}  \mathbf{b} \cos\theta$
	component form?	$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$
To use the angle form of the scalar product, the vectors must be in what orientation?		Tail to tail pointing away from each other
How do you show that two vectors are perpendicular?		Show that <b>a.b = 0</b>
What is <b>a.a</b> equal to? (a dot a)		$ \mathbf{a} ^2$ (the magnitude of a squared)
What is <b>a.(b + c)</b> ?		<b>a.b + a.c</b>

## Relationships and Calculus 1.1 – Polynomials and Quadratics

### a. Quadratics

How do you sketch a parabola?	1. Determine the shape 2. Set the function equal to zero and solve to find the roots (if they exist) 3. Find the y-intercept (by making $x = 0$ ) 4. Find the turning point ( $x$ is halfway between the roots find $y$ by substitution)
How can you find the maximum/ minimum of a quadratic graph from the equation (without differentiation)?	Complete the square

What is the turning point of the graph $y = (x - a)^2 + b$ ?		(a, b)
What is the discriminant?		$b^2 - 4ac$ (b squared minus 4 a c)
What is the discriminant used for?		To determine the nature of the roots of a quadratic equation $b^2 - 4ac > 0$ two distinct, real roots $b^2 - 4ac = 0$ real and equal roots $b^2 - 4ac < 0$ no real roots
If an equation has real and equal roots, what do we know about the discriminant?		$b^2 - 4ac = 0$
If an equation has real and distinct roots, what do we know about the discriminant?		$b^2 - 4ac > 0$
If an equation has real roots, what do we know about the discriminant?		$b^2 - 4ac \geq 0$
If an equation has no real roots, what do we know about the discriminant?		$b^2 - 4ac < 0$
How do you show a line is a tangent to a curve (without differentiation)?		1. Make the line equal the curve 2. Rearrange to make $= 0$ 3. Show that $b^2 - 4ac = 0$ OR 3. Show there is only one point of contact (a repeated root) 4. Make a statement
When a line and a curve are equated	if $b^2 - 4ac > 0$ , then:	The line cuts the curve twice
	if $b^2 - 4ac = 0$ , then:	The line touches the curve once (is a tangent)
	if $b^2 - 4ac < 0$ , then:	The line and curve do not meet
How do you solve a quadratic inequality?		1. Sketch the parabola 2. Write down the range of values for $x$ where the graph is above, $f(x) > 0$ OR below, $f(x) < 0$ the $x$ -axis
<b>b. Polynomials</b>		
How do you find the remainder when a polynomial is divided by $x - a$ ?		Use synthetic division (nested table) with a
How do you show that $(x - a)$ is a factor of $f(x)$ ?		Synthetic division using a to show remainder $= 0$
When factorising a polynomial, what statement must you make?		The remainder is zero so _____ is a factor
When doing synthetic division, what do we call the coefficients along the bottom row?		The quotient

What does the graph of a function with a repeated root look like?	It is a tangent to the $x$ -axis at this root
How do you factorise a cubic function?	<ol style="list-style-type: none"> <li>1. Synthetic division with a factor of the constant</li> <li>2. Use the quotient to make a quadratic</li> <li>3. Factorise the quadratic</li> </ol> You should have 3 brackets!
How do you sketch a polynomial?	<ol style="list-style-type: none"> <li>1. Factorise by synthetic division to find roots</li> <li>2. Find y-intercept</li> <li>3. Differentiate for stationary points and find nature</li> <li>4. Consider large and small values of <math>x</math></li> </ol>

<b>Relationships and Calculus 1.2 – Trigonometric Equations</b>	
How do you solve “simple” trig equations (sin, cos or tan only)?	<ol style="list-style-type: none"> <li>1. Rearrange to get <math>\sin x = \dots</math></li> <li>2. Ignore negatives and use inverse to find the related acute angle</li> <li>3. Use <math>\begin{array}{c c} S &amp; A \\ \hline T &amp; C \end{array}</math> to find the other solution(s)</li> </ol>
If you know the solutions to trig equation between $0^\circ$ and $360^\circ$ , how do you find other solutions?	Add or take away multiples of $360^\circ$
How do you solve double angle trig equations?	<ol style="list-style-type: none"> <li>1. Replace the double angle with the appropriate formula</li> <li>2. Factorise and solve for <math>x</math></li> </ol>
What should you check when answering trig equation questions?	Whether your answer is to be in degrees or radians

<b>Relationships and Calculus 1.3 – Differentiation</b>	
How do you differentiate a function involving powers?	Multiply by the power and decrease the power by one.

How do you prepare to differentiate?		1. Rewrite roots as fractional powers 2. Make sure there are no $x$ terms on the denominator of a fraction 3. Multiply out brackets
What phrases and symbols can be used when discussing differentiation?		1. Rate of change 2. Gradient of the tangent to the curve 3. Derived function 4. $f'(x)$ (f dash $x$ ) 5. $\frac{dy}{dx}$ (dy by dx)
How do you find the derived function, derivative or the rate of change?		Differentiate
How do you find the gradient of a tangent at a particular point?		Differentiate and substitute in the $x$ value at that point
How do you find the equation of a tangent to a curve at a given value of $x$ ?		1. Find the gradient by differentiating and substituting the $x$ value 2. Find the $y$ coordinate by substituting $x$ into the original (not differentiated) function 3. Substitute the gradient and point into $y - b = m(x - a)$
When is a function	Increasing?	When $f'(x) > 0$ (f dash $x$ is greater than zero)
	Decreasing?	When $f'(x) < 0$ (f dash $x$ is less than zero)
	Stationary?	When $f'(x) = 0$ (f dash $x$ is equal to zero)
How do you find the stationary points of a function?		1. Find $f'(x)$ 2. Make $f'(x) = 0$ and solve for $x$ 3. Find the $y$ coordinates by substituting $x$ into $f(x)$ the original function 4. Use a nature table (or the second derivative) to determine their nature
What statement must you write in a stationary point question?		$f'(x) = 0$ for stationary points OR $\frac{dy}{dx} = 0$ for stationary points
What are the four types of stationary point?		1. Maximum 2. Minimum 3. Rising point of inflexion 4. Falling point of inflexion
How do you find where a function is increasing?		1. Differentiate 2. Make $f'(x) > 0$ 3. Solve the inequation

How do you find where a function is increasing?	1. Differentiate 2. Make $f'(x) < 0$ 3. Solve the inequation	
When is a function strictly increasing?	When $b^2 - 4ac$ is always $\geq 0$ (b squared minus 4 a c is always greater than or equal to zero)	
How do you find when a function is strictly increasing?	1. Differentiate 2. Complete the square to show $f'(x)$ is always $\geq 0$	
When is a function strictly decreasing?	When $b^2 - 4ac$ is always $\leq 0$ (b squared minus 4 a c is always less than or equal to zero)	
How do you find when a function is strictly decreasing?	1. Differentiate 2. Complete the square to show $f'(x)$ is always $\leq 0$	
How do you sketch a derived function?	1. Draw axes directly under the original graph 2. Stationary points on the original function are on the $x$ -axis in the derived function 3. If the original function has positive gradient that section is above the $x$ -axis. 4. If the original function has negative gradient that section is below the $x$ -axis 5. Draw a smooth curve that fits all this information.	
What is the derivative of:	$\sin x$ ?	$\cos x$
	$\cos x$ ?	$-\sin x$
What is the derivative of:	$\sin(ax + b)$ ?	$a \cos(ax + b)$
	$\cos(ax + b)$ ?	$-a \sin(ax + b)$
How must angles be measured when integrating or differentiating?	In radians	
How do you differentiate a bracket $(ax + b)^n$ ? (a x plus b to the power of n)	Use the chain rule 1. Multiply by the power 2. Decrease the power by one 3. Differentiate the bracket and multiply by it	

Relationships and Calculus 1.4 – Integration		
How do you integrate a function involving powers?		Increase the power by one, divide by the new power and remember to add C
How do you prepare to integrate?		1. Rewrite roots as fractional powers 2. Make sure there are no $x$ terms on the denominator of a fraction 3. Multiply out brackets
For indefinite integrals always remember:		The constant of integration (+ C)
What is a definite integral?		One where there are limits
What does a definite integral give?		The area under the curve between the limits
What is the integral of:	$\sin x$ ?	$-\cos x + c$
	$\cos x$ ?	$\sin x + c$
What is the integral of:	$\sin(ax + b)$ ?	$\frac{1}{a} \cos(ax + b) + c$
	$\cos(ax + b)$ ?	$\frac{1}{a} \sin(ax + b) + c$
How must angles be measured when integrating or differentiating?		In radians
How do you differentiate a bracket $(ax + b)^n$ ? (a x plus b to the power on n)		Use the chain rule 1. Multiply by the power 2. Decrease the power by one 3. Differentiate the bracket and multiply by it
How do you integrate a bracket $(ax + b)^n$ ? (a x plus b to the power on n)		1. Add one to the power 2. Divide by the new power 3. Differentiate the bracket and divide by it

Applications 1.1 – The Straight line		
What is the gradient of a:	horizontal line?	0
	vertical line?	Undefined
What is the equation of a:	horizontal line?	$y = a$
	vertical line?	$x = a$
What is the formula for the gradient between two points?		$m = \frac{y_2 - y_1}{x_2 - x_1}$ (y two minus y one over x two minus x one)
What is the formula for finding the		$y - b = m(x - a)$

equation of a straight line?	
How do you find the gradient of a straight line from the equation?	1. Rearrange to make $y$ the subject " $y = \dots$ " 2. The gradient is the coefficient of $x$ (the number in front of $x$ )
How do you find the angle a line makes with the positive direction of the $x$ -axis?	$m = \tan \theta$
Gradients of parallel lines are:	Equal
What rule connects the gradients of perpendicular lines?	$m_1 \times m_2 = -1$
If you know the gradient of a line how do you find the gradient of a line perpendicular to it?	Turn it upside down and change the sign
How do you find where two lines meet?	Simultaneous equations
How do you find the midpoint?	$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ x one plus x two over 2, y one plus y two over 2
What is the distance formula?	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ square root of $x$ two minus $x$ one all squared add $y$ two minus $y$ one all squared
What is a perpendicular bisector?	A line that cuts another line in half at right angles
How do you find a perpendicular bisector?	1. Find the midpoint 2. Find the gradient 3. Find the perpendicular gradient 4. Use perpendicular gradient and midpoint in $y - b = m(x - a)$
What is an altitude?	A line through a vertex of a triangle that meets the opposite side at right angles.
How do you find an altitude?	1. Find the gradient of the opposite side 2. Find the perpendicular gradient 3. Use the perpendicular gradient and vertex in $y - b = m(x - a)$
What is a median?	A line through the vertex of a triangle and the midpoint of the opposite side.
How do you find a median?	1. Find the midpoint of the opposite side 2. Find the gradient of the line joining the midpoint and the opposite corner 3. Use this gradient and the opposite vertex in $y - b = m(x - a)$

## Applications 1.2 – The Circle

What is the formula for a circle with: (Formula is on the formula list)	centre (0,0), radius r	$x^2 + y^2 = r^2$
	centre (a,b), radius r	$(x - a)^2 + (y - b)^2 = r^2$
What is the general equation of a circle? (Formula is on the formula list)		$x^2 + y^2 + 2gx + 2fy + c = 0$
From the general equation, how do you find	the centre?	$(-g, -f)$
	the radius?	$\sqrt{g^2 + f^2 - c}$
What does it mean if two circles are congruent?		They have the same radius
What does it mean if two circles are concentric?		They have the same centre
How do you find the equation of a circle?		Find centre and radius and then substitute into $(x - a)^2 + (y - b)^2 = r^2$
How do you show an equation does NOT represent a circle?		1. Look for $x^2 - y^2$ 2. If $r \leq 0$
How do you find where a line and a circle meet?		1. Rearrange the line so that $x =$ or $y =$ 2. Substitute into the circle equation 3. Factorise the quadratic and solve for $x$ or $y$ 4. Find the other variable by substituting the one you have into the equation of the line
How do you show a line is a tangent to a circle?		1. Rearrange the line so that $x =$ or $y =$ 2. Substitute into the circle equation 3. Show that $b^2 - 4ac = 0$ OR 3. Show that there is only one point of intersection 4. Make the appropriate statement
How do you show a line and a circle do not intersect?		1. Rearrange the line so that $x =$ or $y =$ 2. Substitute into the circle equation 3. Show that $b^2 - 4ac < 0$
How do you find the gradient of a tangent to a circle?		1. Find the gradient of the radius 2. The gradient of the tangent is perpendicular to this
How do you find the equation of a tangent to a circle?		1. Find the gradient of the tangent 2. Use a point on the circle (NOT THE



	CENTRE!) and the gradient in $y - b = m(x - a)$
What is a common tangent?	A line that is a tangent to two circles
How do you show that two circles touch eternally?	1. Find the distance between the two centres 2. Find the sum of the two radii 3. Prove distance = $r_1 + r_2$

### Applications 1.3 – Recurrence Relations

When does a recurrence relation have a limit?	When $-1 < a < 1$ (a is between 1 and -1)
How do you find the limit of a recurrence relation?	$L = \frac{b}{1-a}$ (L equals b over 1 minus a)
How do you find the formula of a recurrence relation from three consecutive terms?	1. Make an equation using terms 1 and 2 2. Make an equation using terms 2 and 3 3. Solve simultaneously

### Applications 1.4 – Further Calculus

How do you solve optimisation questions?	Differentiate and find the nature of the stationary points
If you are trying to find a maximum or minimum value in a closed function what do you do?	1. Differentiate to find nature and location of stationary points 2. Ignore any points outside the interval 3. Find $f(x)$ at the limits of the interval (the start and end value) 4. State the answer to the question
How do you find the area underneath a curve?	1. Integrate 2. Substitute in the limits
What do you need to do if asked to find an area?	Sketch the graphs to see if the area is above or below the axis or if it is both
What do you need to remember if finding an area below the $x$ -axis?	The answer will be negative
What do you need to remember if the area is split above and below the $x$ -axis?	1. Calculate the areas above and below the $x$ -axis separately 2. Deal with negative signs 3. Add the areas

How do you find the area between two curves?	Integrate (top curve) – (bottom curve) between the limits
What do you do to the bottom curve when taking it away?	Put it in brackets
How do you find the limits if they are not given?	Find where the curves meet
How do you find where the curves meet?	Make them equal to each other, then solve for $x$
If you know $\frac{dy}{dx}$ how do you find the original equation (y)?	<ol style="list-style-type: none"> <li>1. Integrate to get the general solution</li> <li>2. Substitute in a point to find C and get the particular solution</li> </ol>